Worldline Geometries for Scattering Amplitudes

Based on arXiv:2502.18030, in collaboration with Roberto Bonezzi

RTG 2575:
Rethinking
Quantum Field Theory

HUMBOLDT-UNIVERSITÄT ZU BERLIN

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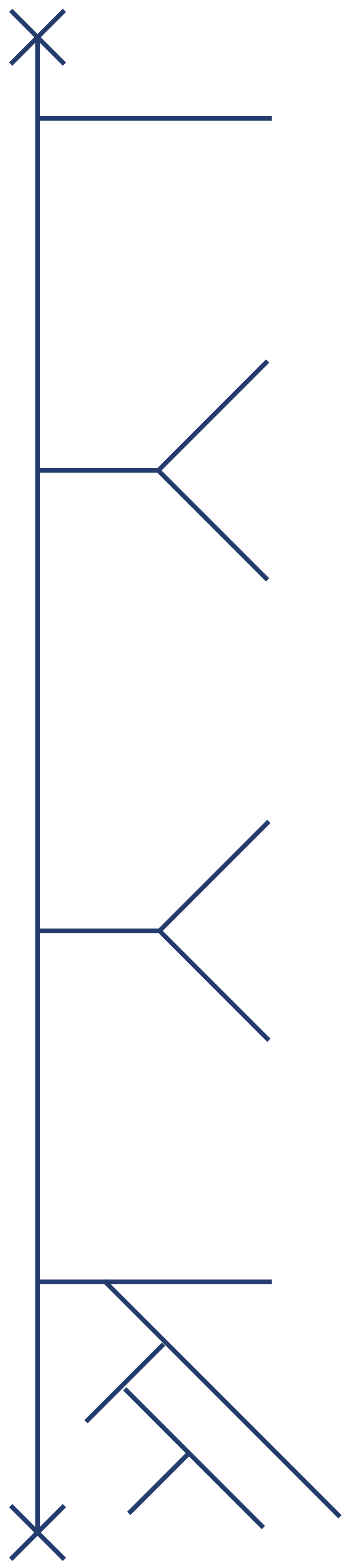
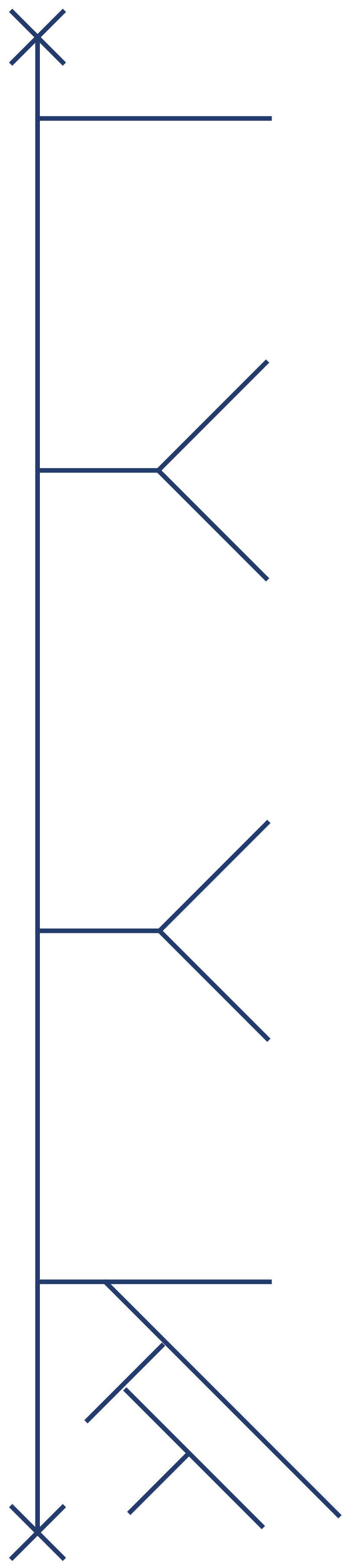
Maria Kallimani, 29.04.2025, Worldliners Seminar

Overview



- Motivation
- Scalar worldline path integrals
- Open worldline geometries
- Multiparticle fields and generalized vertex operators
- General scalar trees
- Summary & Outlook

Overview



Motivation

Scalar worldline path integrals

Open worldline geometries

Multiparticle fields and generalized vertex operators

General scalar trees

Summary & Outlook

Motivation

Goal — Obtain amplitudes in a QFT using ***only*** worldline techniques

Why?

- Better perturbative expansion for **gauge invariance**
- **One** worldline correlator = **many** Feynman diagrams
- String theory-like | **Color-Kinematics?**

Understand **algebraic** structure of gauge theory and gravity — **(Ultimate) Goal**

Motivation

Goal — Obtain amplitudes in a QFT using ***only*** worldline techniques

But…

Worldline correlators

- **Half-ladder** diagrams only
- QFT-based manipulations for **full amplitude**

Endpoint special treatment

$\neq$

Equivalent amplitude external legs

Today — Let’s solve these issues for a **self interacting, scalar theory**!

Overview

- Motivation
- **Scalar worldline path integrals**
- Open worldline geometries
- Multiparticle fields and generalized vertex operators
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- Outlook

Free path integral

Free scalar particle

$$S_0[X,P,e] = \int_0^1 d\tau \left[P_\mu \dot{X}^\mu - e\left(P^2 + m^2\right)\right]$$

Integrate out P

$$S_0[X,e] = \int_0^1 d\tau \left[\frac{1}{4e}\dot{X}^2 - em^2\right]$$

Diffeomorphism invariance

$$\delta X^\mu = \xi \dot{X}^\mu$$
$$\delta e = \partial_\tau(e\xi)$$

BRST quantization

Ghosts

$$S_{gh}[b,c] = \int_0^1 d\tau\, b\dot{c}$$

Free path integral

Worldline

$\tau = 0$ $\tau = 1$

Boundary conditions

Dirichlet-Dirichlet (DD)

$X^\mu(0) = x^\mu \qquad \xi^\mu(0) = \xi^\mu(1) = 0$

$X^\mu(1) = y^\mu \qquad c(0) = c(1) = 0$

Dirichlet-Neumann (DN)

$X^\mu(0) = x^\mu \qquad \xi^\mu(0) = 0$

$\dot{X}^\mu(1) = k^\mu \qquad c(0) = 0 \qquad b(1) = 0$

Neumann-Neumann (NN)

$\dot{X}^\mu(0) = k^\mu \qquad \xi^\mu(0) = \xi^\mu(1)$

$\dot{X}^\mu(1) = k'^\mu \qquad b(0) = b(1) = 0$

Free path integral

Worldline

$\tau = 0$ $\tau = 1$

Endpoint with Dirichlet BC

Boundary conditions

Dirichlet-Dirichlet (DD)

$X^\mu(0) = x^\mu$ $\quad \xi^\mu(0) = \xi^\mu(1) = 0$

$X^\mu(1) = y^\mu$ $\quad c(0) = c(1) = 0$

- No Killing vector
- Gauge invariant modulus $\quad T := \int_0^1 d\tau e(\tau)$
- Gauge fixing $\quad e = T$

Free path integral

Worldline

$\tau = 0$ — $\tau = T$ Rescale

Boundary conditions

Dirichlet-Dirichlet (DD)

$X^\mu(0) = x^\mu$ $\quad \xi^\mu(0) = \xi^\mu(T) = 0$

$X^\mu(T) = y^\mu$ $\quad c(0) = c(T) = 0$

- No Killing vector
- Gauge invariant modulus $\quad T := \int_0^T d\tau e(\tau)$
- Gauge fixing $\quad e = 1$

Free path integral

$$Z_0(x,y) = \int \frac{\mathcal{D}X\mathcal{D}e}{\text{VolGauge}} e^{iS[X,e]} = \int_0^\infty dT \int \mathcal{D}X e^{iS[X,1]} \int \mathcal{D}c\mathcal{D}b e^{iS_{gh}}$$

Can set to 1

Use trajectory:

$$X^\mu(\tau) = x^\mu + (y^\mu - x^\mu)\frac{\tau}{T} + z(\tau)$$

classical solution

fluctuations $z^\mu(0) = z^\mu(T) = 0$

Mass factors out as overall phase

$$Z_0(x,y) = \int_0^\infty dT e^{-im^2T} K_0(x,y;T)$$

$$K_0(x,y;T) = \int [\mathcal{D}X]_x^y \exp\left(i\int_0^T d\tau \frac{\dot{X}^2}{4}\right) = \frac{e^{-i\frac{(x-y)^2}{4T}}}{(4\pi i T)^{D/2}}$$

$$Z_0(x,y) \equiv -i\int \frac{d^Dp}{(2\pi)^D} \frac{e^{ip\cdot(x-y)}}{p^2+m^2-i\epsilon} = D_0(x,y)$$

Scalar field theory propagator (position space)

Background field

Particle in potential

$$S_{\mathscr{V}}[X] = S_0[X,1] - \int_0^T d\tau \mathscr{V}(X)$$

Generated by **background field**

$$S_{\text{FT}}[\phi] = \int d^D x \Big[-\frac{1}{2}\partial^\mu \phi \partial_\mu \phi - \mathscr{U}(\phi) \Big]$$

$$\mathscr{V}(X) \equiv \mathscr{U}''(\phi(X))$$

Field Theory propagator

$$D(x, y; \phi) = \langle y | \frac{i}{\Box - \mathscr{U}''(\phi)} | x \rangle = \int_0^\infty dT \langle y | e^{-i(P^2 - \mathscr{U}''(\phi))} | x \rangle \equiv Z_{\mathscr{V}}(x, y)$$

Vertex Operators

Evaluate $Z_{\mathscr{V}}$ perturbatively, with **plane wave** field configuration

$$\phi(x) = \sum_{i=1}^{n} e^{ik_i \cdot x}$$

Vertex Operators

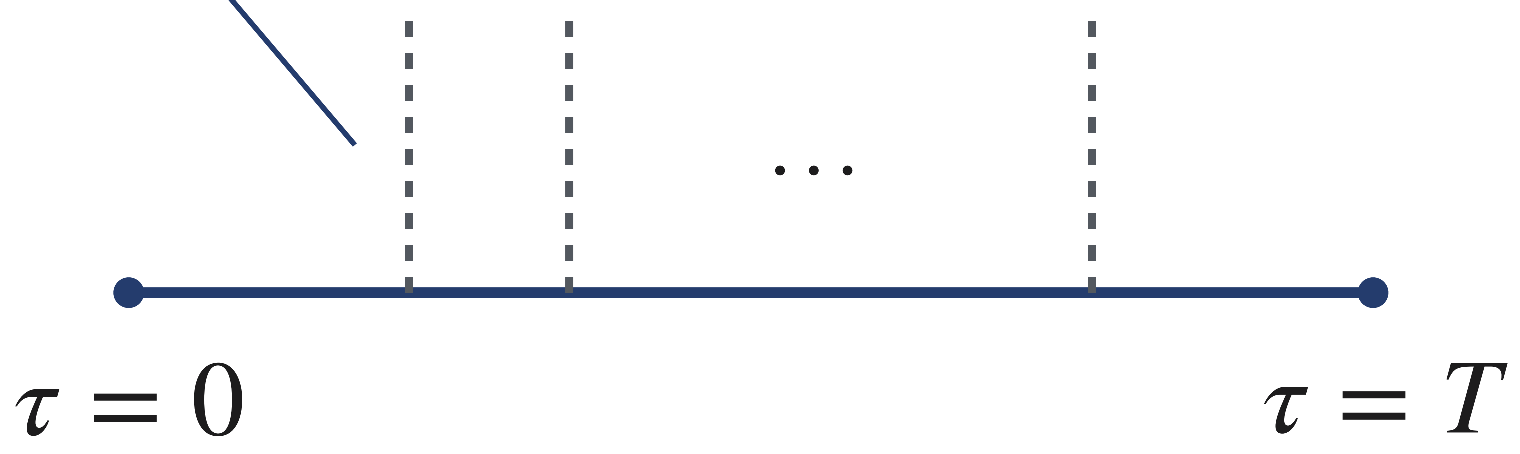
$$V_i(\tau) := -\, i\,\mathscr{U}''(e^{ik_i \cdot X(\tau)})$$

e.g. ϕ^3 theory:

$$\mathscr{U}(\phi) = \frac{g}{3!}\phi^3$$

$$V_i(\tau) := -\, ige^{ik_i \cdot X(\tau)}$$

Insert into worldline PI



Integrate over insertion position

$$Z_{\mathscr{V}}(x,y) = \sum_{n=0}^{\infty} \frac{1}{n!} D_n(x,y;\{k_i\}_{i=1}^{n})$$

Dressed propagator

Normalized averages

$$Z_{\mathscr{V}}(x,y) = \sum_{n=0}^{\infty} \frac{1}{n!} D_n(x,y;\{k_i\}_{i=1}^{n})$$

Dressed propagator

$$D_n(x,y;\{k_i\}_{i=1}^{n}) = \int_0^{\infty} dT e^{-im^2T} K_0(x,y;T) \sum_{n=0}^{\infty} \frac{(-i)^n}{n!} \left(\prod_{k=0}^{n} \int_0^T d\tau_k \right) \left\langle V(\tau_1)\ldots V(\tau_k) \right\rangle_{\mathrm{DD}}$$

Normalized average

$$\left\langle F[z] \right\rangle_{DD} := (4\pi i T)^{D/2} \int [\mathscr{D}z]_0^0 \, F[z] \, \exp\left\{ i \int_0^T d\tau \frac{\dot{z}^2}{4} \right\}$$

$$\left\langle 1 \right\rangle_{DD} = 1$$

Free path integral

Worldline

$\tau = 0$ $\tau = T$

Boundary conditions

Neumann-Neumann (NN)

$\dot{X}^\mu(0) = k^\mu$ $\xi^\mu(0) = \xi^\mu(T)$

$\dot{X}^\mu(T) = k'^\mu$ $b(0) = b(T) = 0$

Change symplectic structure

$$S_P[X, P] = -\int_0^T d\tau \left[\dot{P}_\mu X^\mu + P^2 + m^2\right]$$

$P^\mu(0) = k^\mu$

$P^\mu(T) = k'^\mu$

How to define a momentum space path integral?

Momentum space path integral

Non-relativistic PI ⟶ Transition amplitudes between momentum eigenstates

$$\langle k' | e^{-iTP^2} | k \rangle \equiv \int \mathscr{D}X[\mathscr{D}P]_k^{k'} \exp(iS_P[X,P]) = (2\pi)^D \delta^D(k-k') e^{-iTk^2}$$

$$X^\mu(\tau) = x_0^\mu + z^\mu(\tau)$$

$$\int_0^T d\tau\, z^\mu(\tau) = 0$$

Momentum conservation from zero mode

$$= (2\pi)^D \delta^D(k-k') \int \mathscr{D}z[\mathscr{D}P]_k^{k'} \exp(iS_P[z,P])$$

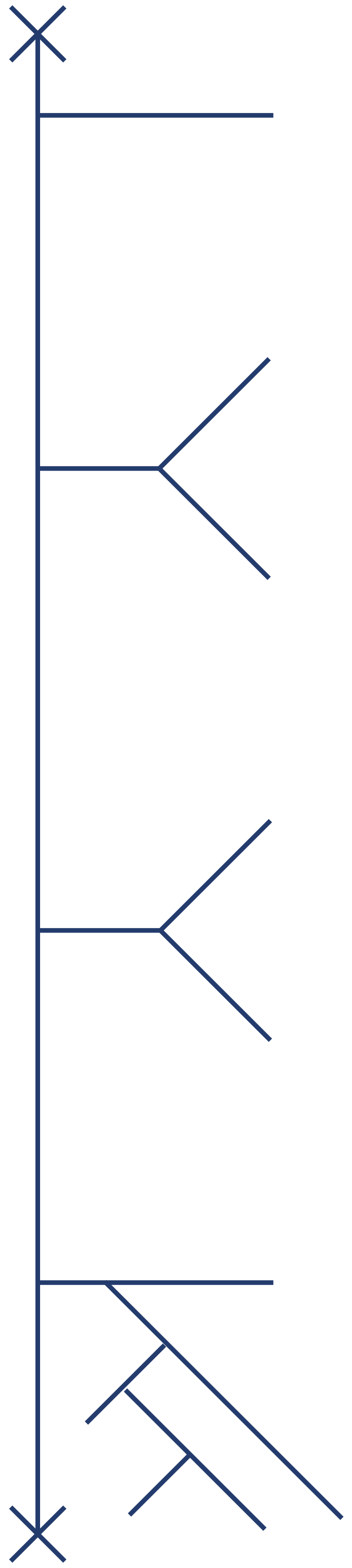
$$\Rightarrow \int \mathscr{D}z[\mathscr{D}P]_k^{k} \exp(iS_P[z,P]) = e^{-iTk^2}$$

$k = 0$

Normalized average

$$\langle F[z] \rangle_{NN} := \int \mathscr{D}z[\mathscr{D}P]_0^0\, F[z] \exp\left\{ i \int_0^T d\tau \frac{\dot{z}^2}{4} \right\}$$

Overview



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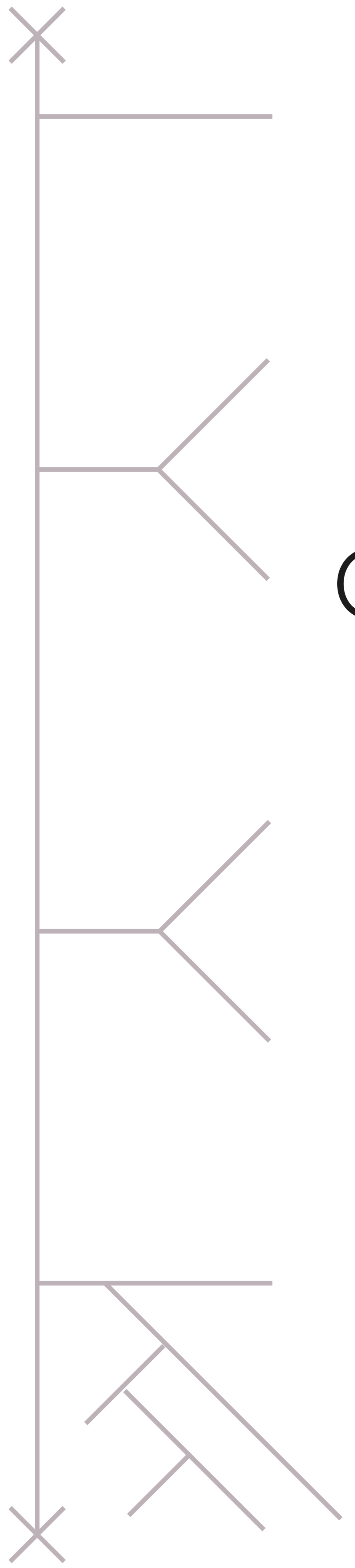
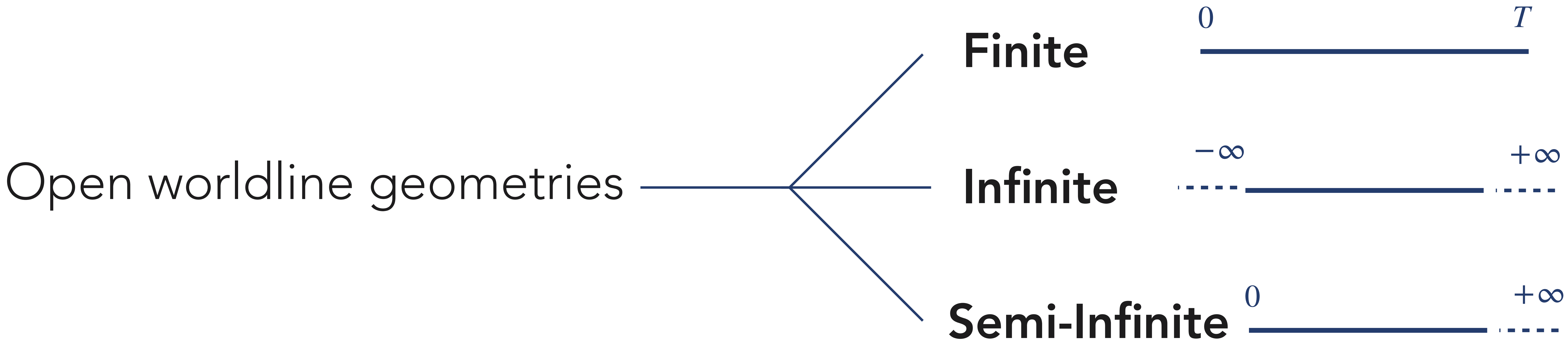




Open worldline geometries
Finite
0
T
Infinite
$-\infty$
$+\infty$
Semi-Infinite
0
$+\infty$

Finite Line: Dressed Propagators

*Only needed at loop level

Path integral on finite line ⟶ Dressed propagators

DD Generating Functional

$$Z_{\mathrm{DD}}[j] := = \frac{1}{(4\pi i T)^{D/2}} \left\langle e^{i\int_0^T d\tau j(\tau)\cdot z(\tau)} \right\rangle_{\mathrm{DD}}$$

NN Generating Functional

$$Z_{NN}[j] = (2\pi)^D \delta^D(j_0) \left\langle e^{i\int_0^T d\tau\, j'(\tau)\cdot z(\tau)} \right\rangle_{\mathrm{NN}}$$

Zero mode of source: momentum conservation

No zero mode

$$\left\langle e^{i\int_0^T d\tau j(\tau)\cdot z(\tau)} \right\rangle_{\mathrm{BC}} = \exp\left\{ \frac{i}{2} \int_0^T d\tau d\sigma\, j^\mu(\tau)\, G_{\mathrm{BC}}(\tau,\sigma)\, j_\mu(\sigma) \right\}$$

$$G_{\mathrm{DD}}(\tau,\sigma) = |\tau-\sigma| - (\tau+\sigma) + \frac{2}{T}\tau\sigma$$

$$G_{\mathrm{NN}}(\tau,\sigma) = |\tau-\sigma| - \frac{1}{T}(\tau^2+\sigma^2) + \tau + \sigma - \frac{2T}{3}$$

NN and the infinite line

Action for $\tau \in (-\infty, \infty)$

$$S_\infty[X, e] = \int_{-\infty}^{+\infty} d\tau \left[\frac{1}{4e}\dot{X}^2 - em^2\right]$$

NN →

$$\dot{X}^\mu(\pm\infty) = 0$$

$$\xi^\mu(\infty) = \xi^\mu(-\infty)$$

$$e(\infty) = e(-\infty) = 1$$

Constant Killing vector:
Translation invariance

$c(\infty) = c(-\infty)$
Can have zero mode

$$T = \int_{-\infty}^{+\infty} d\tau\, e(\tau) \to \infty$$

No gauge invariant modulus!

$$Z_\infty(x, y) = \int \frac{\mathscr{D}X\mathscr{D}e}{\text{VolGauge}} e^{iS[X,e]} = \int \mathscr{D}X e^{iS[X,1]} \int \mathscr{D}c\mathscr{D}b e^{iS_{\text{gh}}}$$

Saturate c zero mode:
Fix position of one operator

Can set to 1

NN and the infinite line

Normalized Average

$$\left\langle F[z] \right\rangle_{NN}^{\infty} := \int [\mathscr{D}X]_{NN}^{0} F[z] \exp\left\{ iS_{\infty}[X,1] \right\}$$

Generating Functional

$$Z_{NN}^{\infty}[j] = (2\pi)^D \delta^D(j_0) \left\langle e^{i\int_0^T d\tau\, j'(\tau)\cdot z(\tau)} \right\rangle_{\text{NN}}^{\infty}$$

Zero mode of source: momentum conservation

No zero mode

$$\left\langle e^{i\int_0^T d\tau\, j'(\tau)\cdot z(\tau)} \right\rangle_{\text{NN}}^{\infty} = = (2\pi)^D \delta^D(j_0) \lim_{T\to\infty} \exp\left\{ \frac{i}{2} \int_{-T/2}^{T/2} d\tau d\sigma\, j^{\mu}(\tau) G_{\text{NN}}^{\text{shift}} j_{\mu}(\sigma) \right\}$$

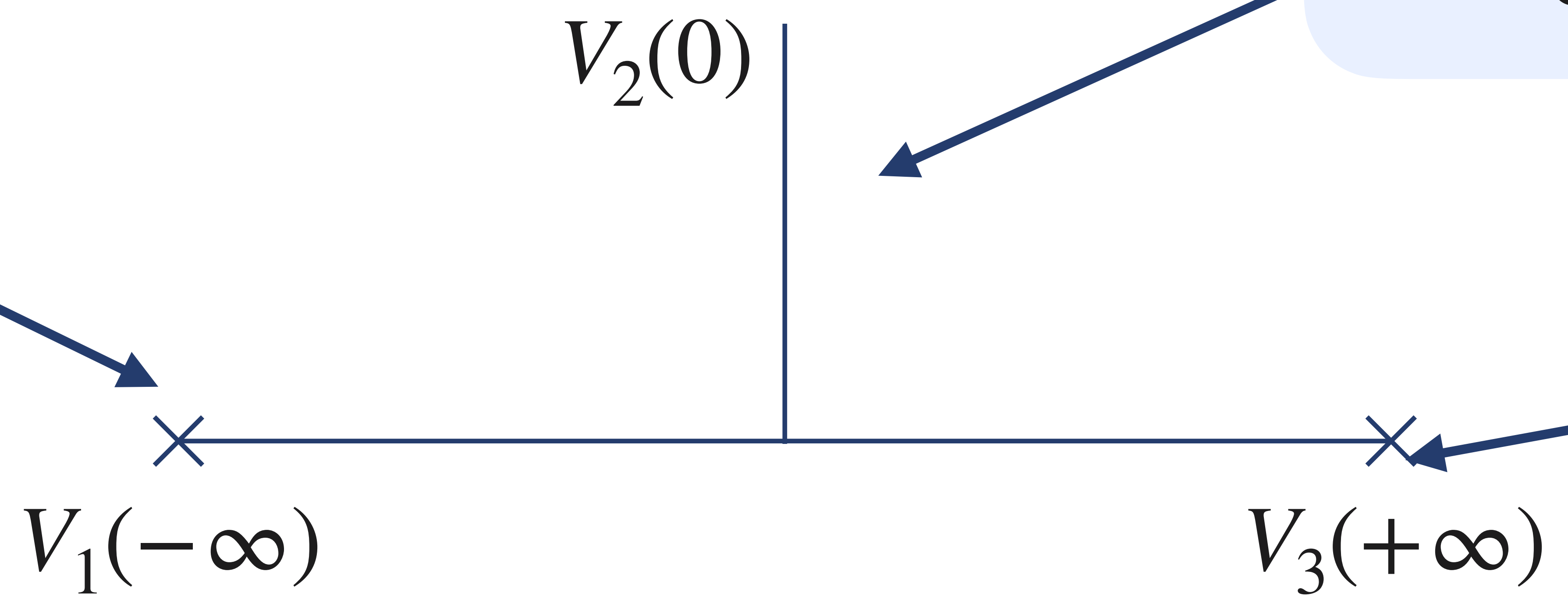
$$\lim_{T\to\infty} G_{\text{NN}}^{\text{shift}}(\tau,\sigma) = |\tau - \sigma|$$

Infinite line: Amplitudes

Example: 3-point

Insert VOs for external legs: one can be fixed

Insert VOs to create asymptotic states



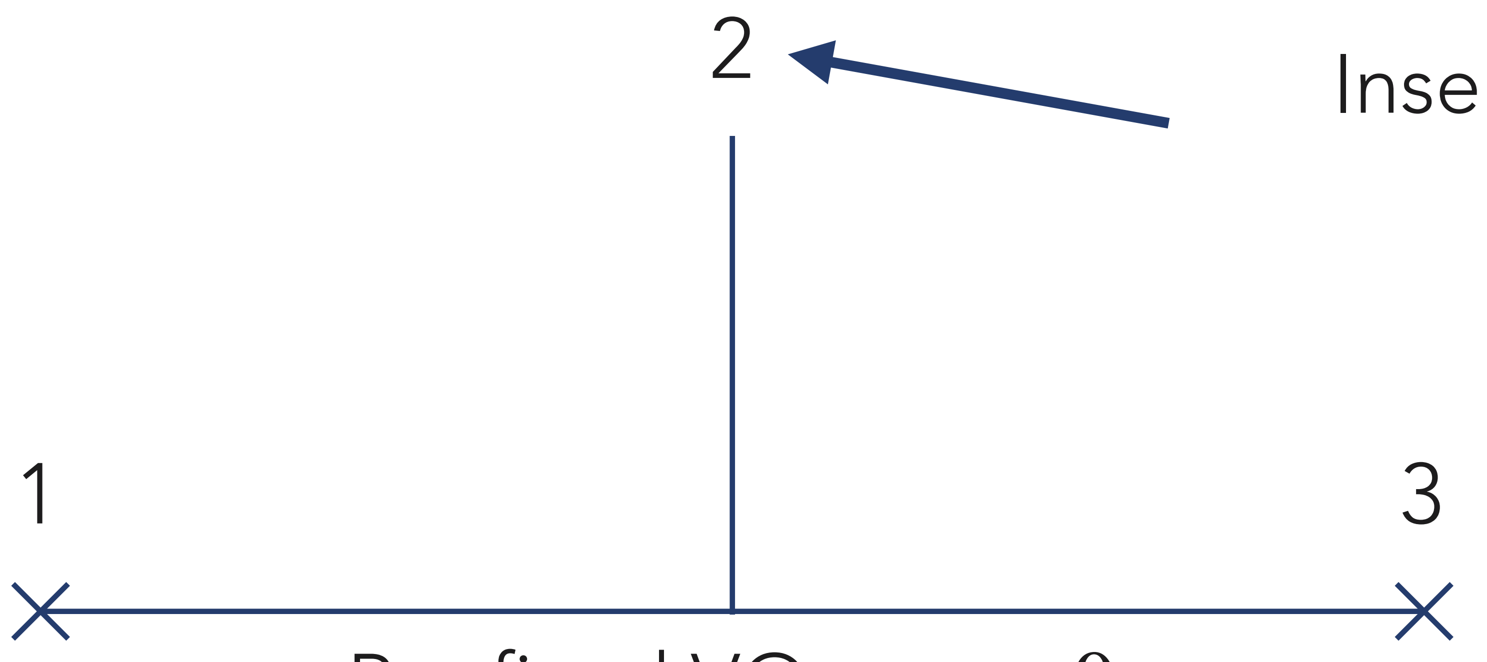
Infinite NN worldline zero momentum endpoints

Multiply by coupling, no real vertex at endpoints

$$\mathscr{A}_3 = (-ig)^{-2} e^{-im^2 T} (2\pi)^D \delta^D(k_1 + k_2 + k_3) \left\langle\!\!\left\langle V_1(-T/2)\, V_2(0)\, V_3(+T/2) \right\rangle\!\!\right\rangle \Big|_{T\to\infty}$$

$$\left\langle\!\!\left\langle V_1(\tau_1)\cdots V_N(\tau_N) \right\rangle\!\!\right\rangle := \left\langle e^{ik_1\cdot z(\tau_1)} \ldots e^{ik_N\cdot z(\tau_N)} \right\rangle^{\infty}_{\mathrm{NN}}$$

LSZ reduction



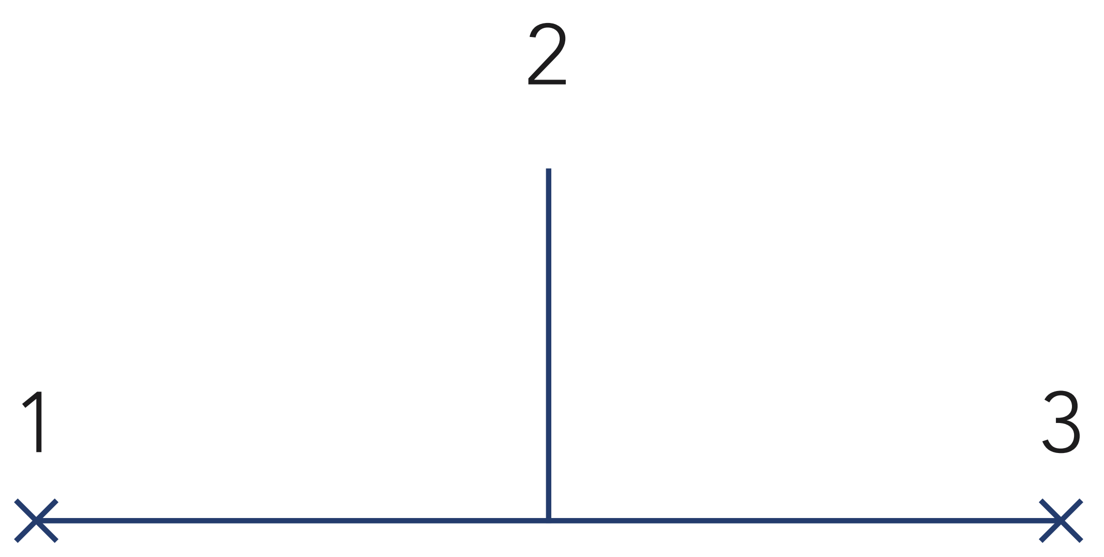
$$e^{-im^2T}\left\langle\!\left\langle V_1(-T/2)\, V_2(0)\, V_3(+T/2) \right\rangle\!\right\rangle = e^{-i(T/2+\tau_*)(k_1^2+m^2)-i(T/2-\tau_*)(k_3^2+m^2)}$$

$$e^{-i\frac{T}{2}(k^2+m^2-i\epsilon)} = \begin{cases} 0, & k^2+m^2 \neq 0 \\ 1, & k^2+m^2 = 0 \end{cases}$$

$T \to \infty$
implements **LSZ reduction**!

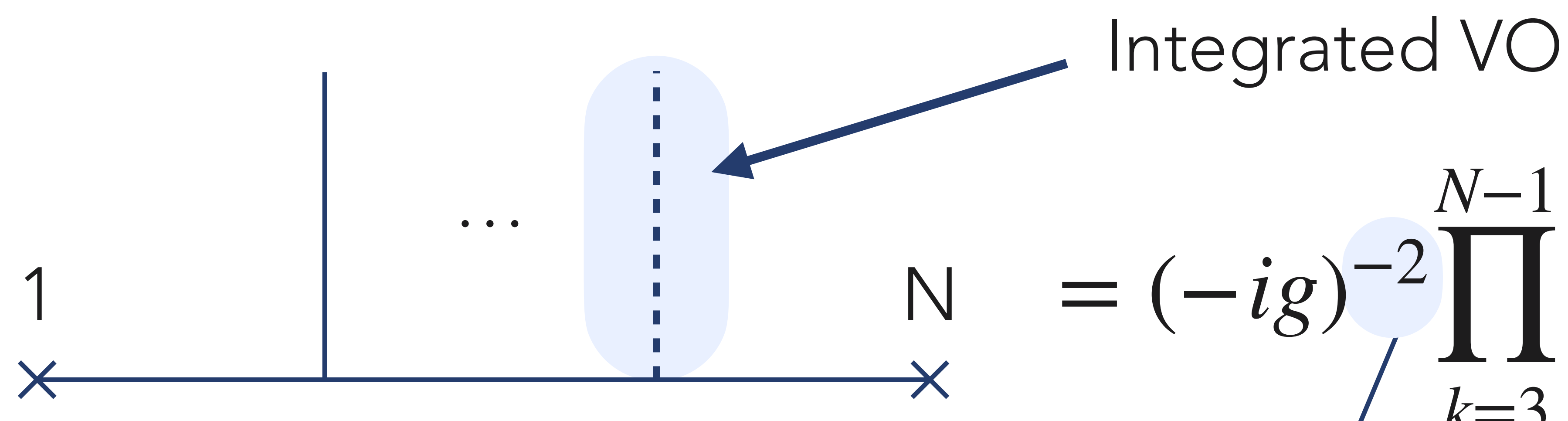
Amplitudes

$\mathscr{A}_3 =$



$= -ig$

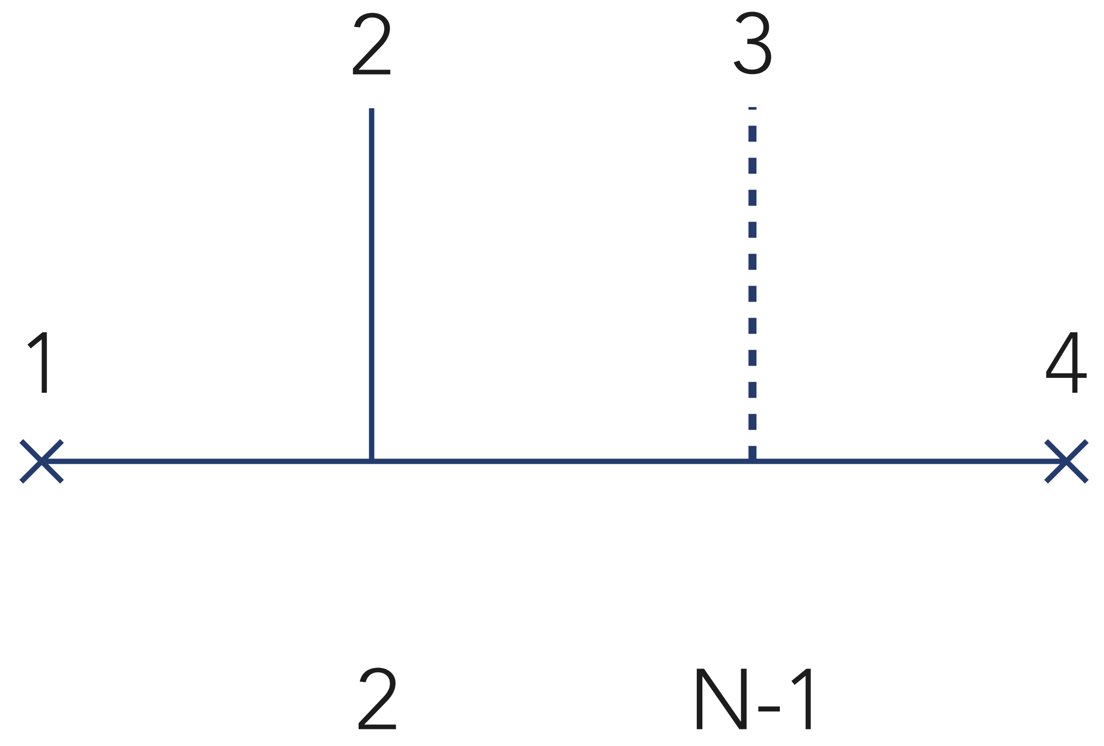
$\mathscr{A}_N^{\text{ladder}} =$



$$= (-ig)^{-2} \prod_{k=3}^{N-1} \int_{-\infty}^{+\infty} d\tau_k \left\langle\!\!\left\langle V_1(-\infty)\, V_2(0) \prod_{i=3}^{N-1} V_i(\tau_i)\, V_N(+\infty) \right\rangle\!\!\right\rangle$$

~closed string: sphere

$\mathscr{A}_4 \supset$

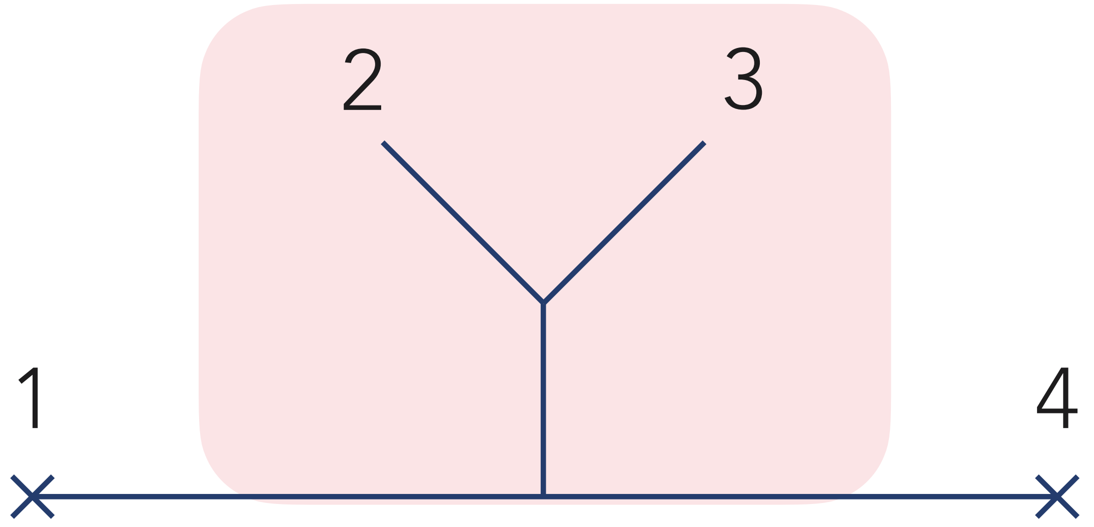


$$= (-ig)^{-2} \int_{-\infty}^{+\infty} d\tau_* \left\langle\!\!\left\langle V_1(-\infty) V_2(0) V_3(\tau_*) V_4(+\infty) \right\rangle\!\!\right\rangle = \frac{ig^2}{s+m^2} + \frac{ig^2}{u+m^2}$$

Where is the missing channel?

A bilinear vertex operator

Missing channel Requires insertion of VO coming from **two-particle field**!



$$= (-ig)^{-2} \left\langle\!\left\langle V_1(-\infty) V_{23}(0) V_4(+\infty) \right\rangle\!\right\rangle = \frac{ig}{t^2 + m^2}$$

$$V_{23}(\tau) := -ig\phi_{23}(X(\tau))$$

Generalized Vertex Operator (gVO)

$$(\Box - m^2)\phi_{23} = g\phi_2\phi_3$$

Goal Obtain generalized vertex operators as worldline path integrals

DN and the semi-infinite line

Action for $\tau \in [0,\infty)$

$$S_{SI}[X,e] = \int_0^{+\infty} d\tau \left[\frac{1}{4e} \dot{X}^2 - em^2 \right]$$

DN →

$X^\mu(0) = x^\mu$

$\xi(0) = 0$

$c(0) = 0$

$\dot{X}^\mu(\infty) = 0$

$e(\infty) = 1$

$b(\infty) = 0$

No Killing vector

$T \to \infty$

No gauge invariant modulus!

$$Z_{SI}(x,y) = \int [\mathscr{D}X]_{DN} e^{iS[X,1]} \int \mathscr{D}c\mathscr{D}b e^{iS_{gh}}$$

Can set to 1

DN Generating functional

Use trajectory: $X^\mu(\tau) = x^\mu + z^\mu(\tau)$ $\quad z^\mu(0) = 0$, $\quad \dot{z}^\mu(\infty) = 0$

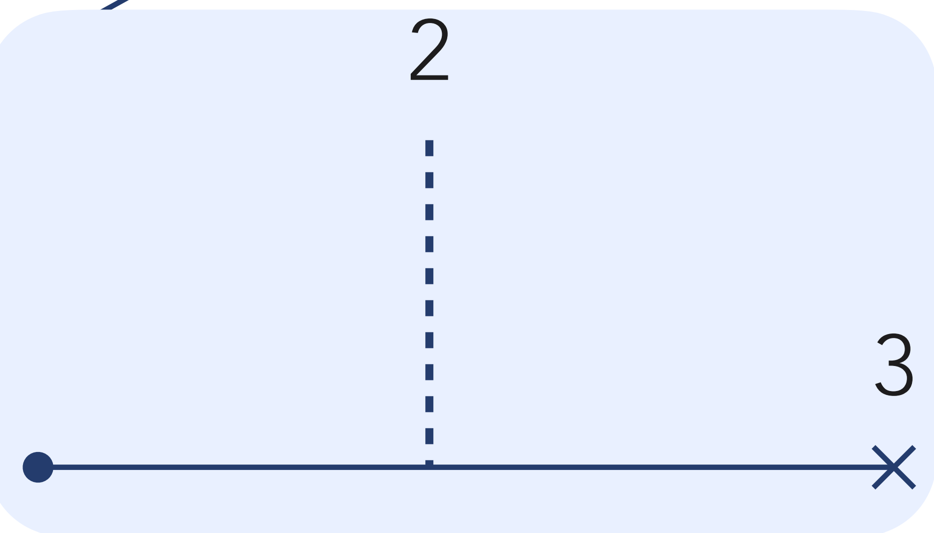
DN Generating Functional

$$Z_{\mathrm{DN}}[j] \equiv \left\langle \exp\, i \int_0^\infty d\tau\, j_\mu(\tau)\, X^\mu(\tau) \right\rangle_{\mathrm{DN}}^{\infty} = e^{i j_0 \cdot x} \exp\left\{ \frac{i}{2} \int_0^\infty d\tau d\sigma \left(j^\mu(\tau)\, G_{\mathrm{DN}}(\tau,\sigma)\, j_\mu(\sigma) \right) \right\}$$

$$G_{\mathrm{DN}}(\tau,\sigma) = |\tau - \sigma| - (\tau + \sigma)$$

Get generalized VOs

$$V_{23}(\tau) = (-ig)^{-1}$$

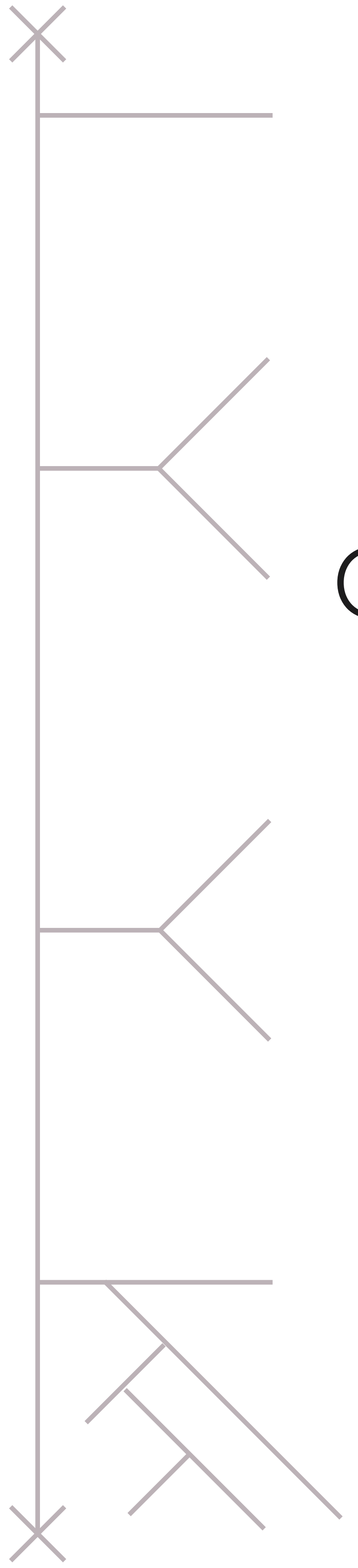


Bilinear field ϕ_{23}

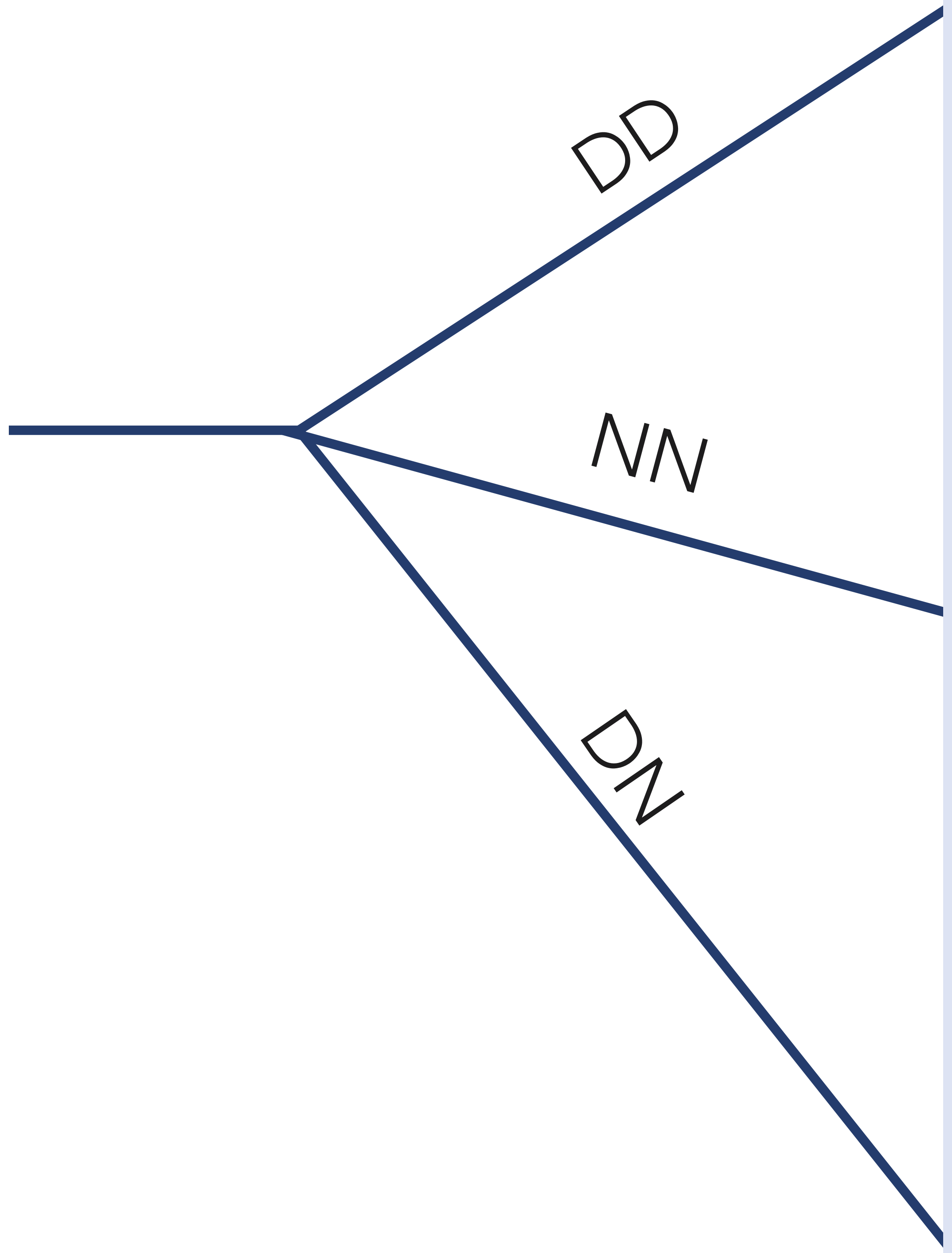
$$= (-ig)^{-1} \int_0^\infty d\tau \left\langle V_2(\tau) V_3(+\infty) \right\rangle_{\mathrm{DN}}^{\infty} = -\frac{g}{(k_2+k_3)^2 + m^2}\, e^{i(k_2+k_3)\cdot x} \Big|_{x = X(\tau)}$$

~open string: disk

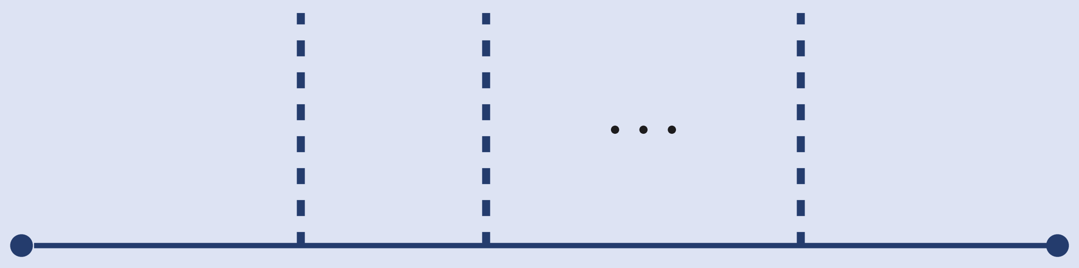
Overview



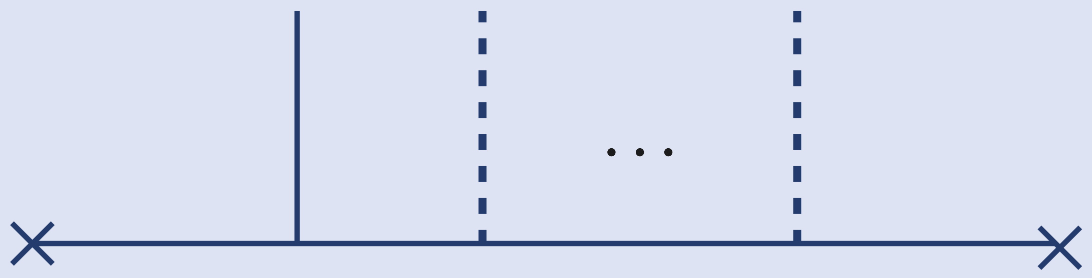
Open worldline geometries



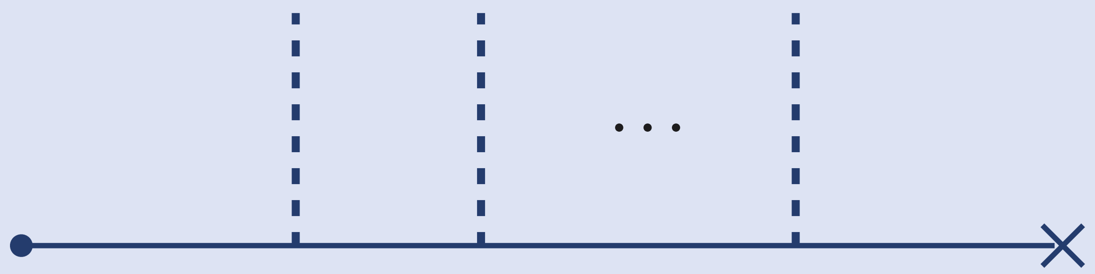
Finite | Dressed Propagators



Infinite | Amplitudes



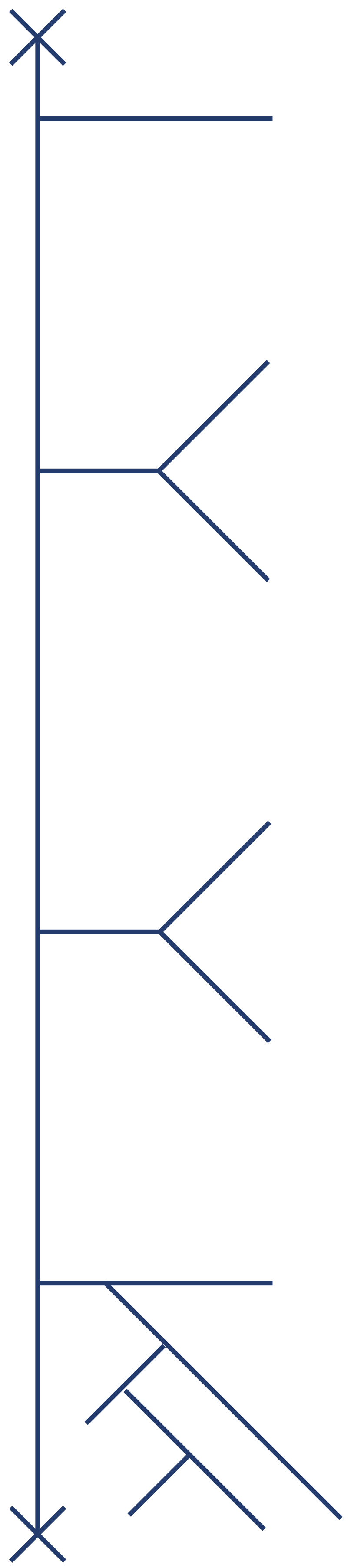
Semi-Infinite | Vertex Operators



Worldline geometries

Geometry	Boundary Conditions	Object	Prescription	Generating functional
Finite	Dirichlet-Dirichlet	Dressed Propagators	• Set $e = T$ • Integrate over T • All vertex operators integrated	$Z_{\rm DD}[j] := \exp\left\{\frac{i}{2}\int_0^T d\tau d\sigma\, j^\mu(\tau)\, G_{\rm DD}(\tau,\sigma)\, j_\mu(\sigma)\right\}$ $G_{\rm DD}(\tau,\sigma) = \lvert\tau-\sigma\rvert - (\tau+\sigma) + \frac{2}{T}\tau\sigma$
Infinite	Neumann-Neumann	(Tree level) Amplitudes	• Set $e = 1$ • Take $T \to \infty$ • Fix one vertex operator	$Z^{\infty}_{\rm NN}[j] := \exp\left\{\frac{i}{2}\int_{-\infty}^{+\infty} d\tau d\sigma\, j^\mu(\tau)\,\lvert\tau-\sigma\rvert\, j_\mu(\sigma)\right\}$
Semi-Infinite	Dirichlet-Neumann	Multiparticle Fields/ Generalized Vertex Operators	• Set $e = 1$ • Take $T \to \infty$ • All vertex operators integrated	$Z_{\rm DN}[j] := \exp\left\{\frac{i}{2}\int_0^T d\tau d\sigma\, j^\mu(\tau)\, G_{\rm DN}(\tau,\sigma)\, j_\mu(\sigma)\right\}$ $G_{\rm DN}(\tau,\sigma) = \lvert\tau-\sigma\rvert - (\tau+\sigma)$

Overview



Motivation

Scalar worldline path integrals

Open worldline geometries

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General scalar trees

Summary & Outlook

Perturbative solutions

ϕ^3 theory (massless)

$$\mathcal{U}(\phi) = \frac{g}{3!}\phi^3$$

EOM

$$\Box\phi = \frac{g}{2}\phi^2$$

Solve perturbatively

$$\phi(x) = \sum_{n=1}^{\infty}\frac{1}{n!}\phi^{(n)}(x)$$

$$\Box\phi^{(1)} = 0$$

$$\Box\phi^{(2)} = g\phi^{(1)}\phi^{(1)}$$

$$\phi^{(n)} = \frac{1}{\Box}\frac{g}{2}\sum_{k+l=n}\binom{n}{k}\phi^{(k)}\phi^{(l)}$$

Propagator

(Unpolarized) gVO

$$V^{(n)}(\tau) := -ig\phi^{(n)}(X(\tau))$$

Polarize

$$\phi^{(1)}(x) = \sum_{r=1}^{n}\phi_r(x) = \sum_{r=1}^{n}e^{ik_r\cdot x}$$

$$\phi_{12\ldots n} = \frac{1}{\Box}\frac{g}{2}\sum_{k+l=n}\binom{n}{k}\phi^{(k)}\phi^{(l)}\Big|_{12\ldots\mathrm{n}}$$

(Polarized) gVO

$$V_{12\ldots n}(\tau) := -ig\phi_{12\ldots n}(X(\tau))$$

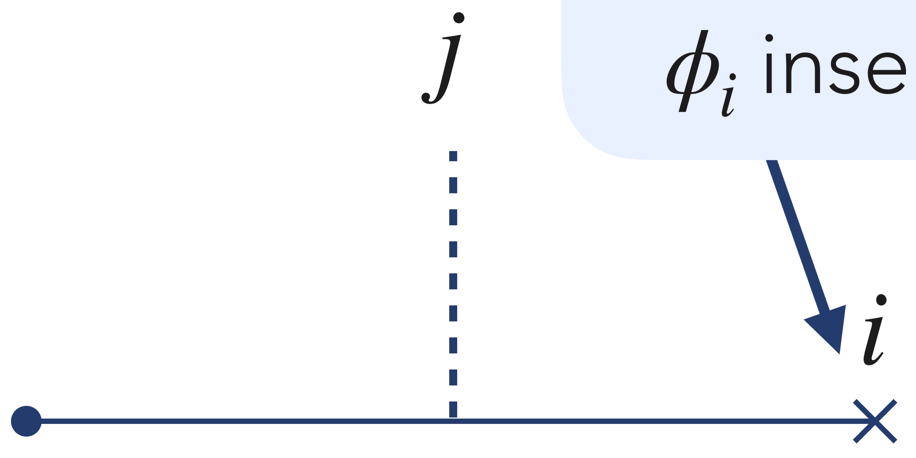
As worldline correlators

$\phi_i(x) = (-ig)^{-1}$

$= \left\langle e^{ik_i\cdot X(\infty)} \right\rangle_{DN}^{\infty} = e^{ik_i\cdot x} \longrightarrow V_i(\tau) = -ig\phi_i(X(\tau))$

From here on: ϕ_i insertion

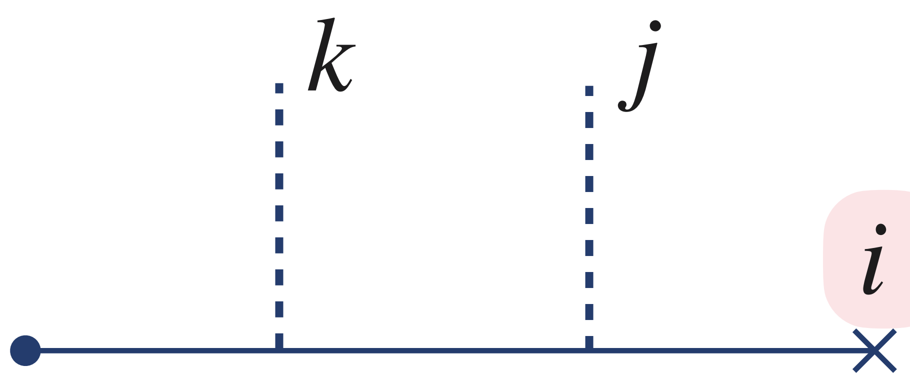
$\phi_{ij}(x) =$



$\bar{V}_j$

$= \int_0^\infty d\tau \left\langle V_j(\tau)\phi_i(X(\infty)) \right\rangle_{DN}^{\infty} = -g\frac{e^{ik_{ij}\cdot x}}{(k_i+k_j)^2} \longrightarrow V_{ij}(\tau) = -ig\phi_{ij}(X(\tau))$

$\phi_{ijk}(x) =$



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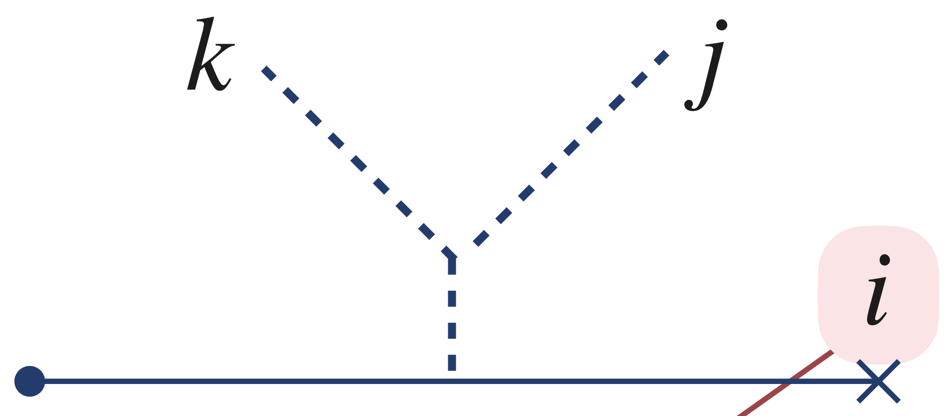
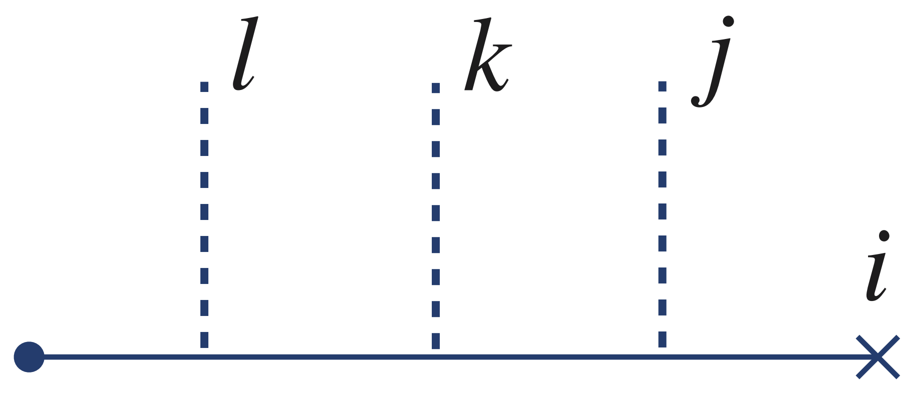


$\longrightarrow V_{ijk}(\tau) = -ig\phi_{ijk}(X(\tau))$

Choose **any** state to be the one at infinity, but **same** for all diagrams

As worldline correlators

$\phi_{ijkl}(x) =$



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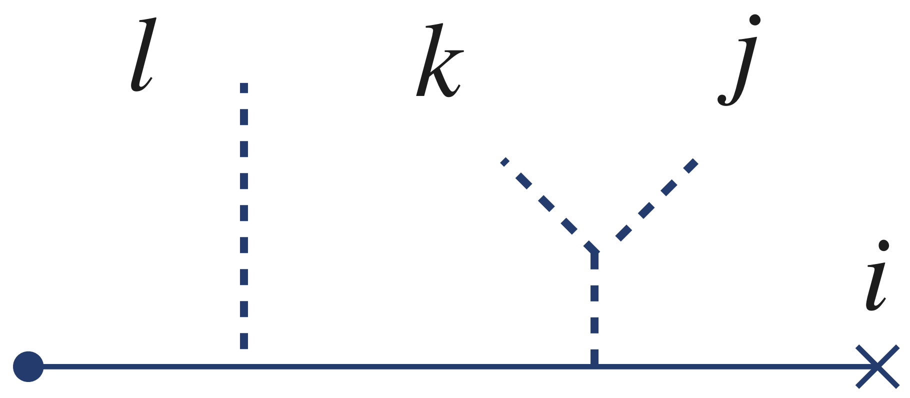
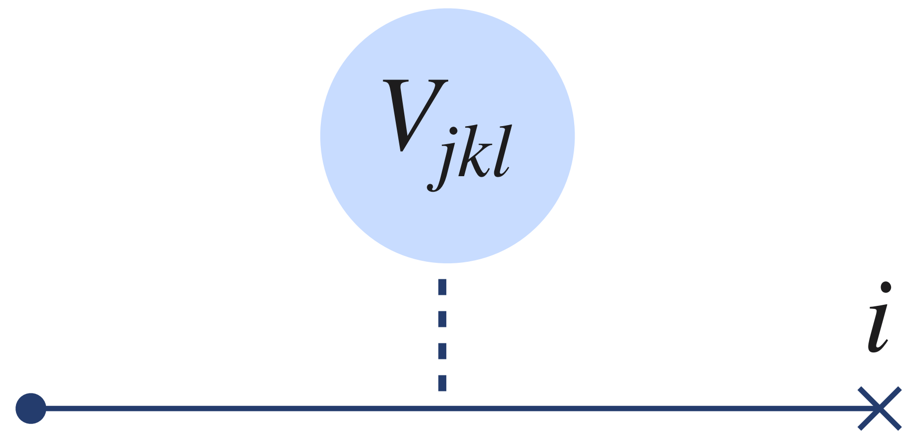


$+$ 2 more

$+$



$$V_{ijkl}(x) = -ig\phi_{ijkl}(X(\tau))$$

A recursive relation

Unpolarize: all legs are identified, obtain:

Create boundary state with $-igV^{(1)}(X(\infty))$

$$\sum_{m=1}^{\infty} \frac{1}{(m-1)!}\phi^{(m)}(x) = \left\langle \exp\left(\sum_{k=1}^{\infty} \frac{1}{k!}\overline{V}^{(k)}\right) \phi^{(1)}(X(\infty)) \right\rangle_{\mathrm{DN}}^{\infty}$$

➔ **Recover same multilinear fields as from perturbative solutions**

Obtain gVOs of any order by polarizing:

$$\phi^{(1)}(x) = \sum_{r=1}^{n} \phi_r(x) = \sum_{r=1}^{n} e^{ik_r\cdot x}$$

$$\bar{V}^{(k)}(x) = \bar{V}_{(2+3+\ldots+n)(2+3+\ldots+n)\ldots(2+3+\ldots+n)}$$

$$(1+2)(1+2) = 12$$

$$(1+2)3 = 13 + 23$$

Overview

Motivation

Scalar worldline path integrals

Open worldline geometries

Multiparticle fields and generalized vertex operators

General scalar trees

Summary & Outlook

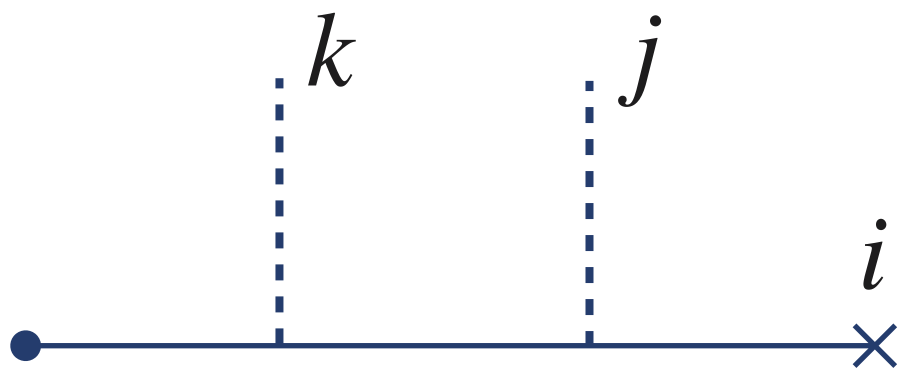
Adding more self interactions

$\phi^3 + \phi^4$ theory (massless)

$$\mathcal{U}(\phi) = \frac{g}{3!}\phi^3 + \frac{\lambda}{4!}\phi^4$$

gVOs now **not** simply proportional to fields!

$\phi_{ijk}(x) =$



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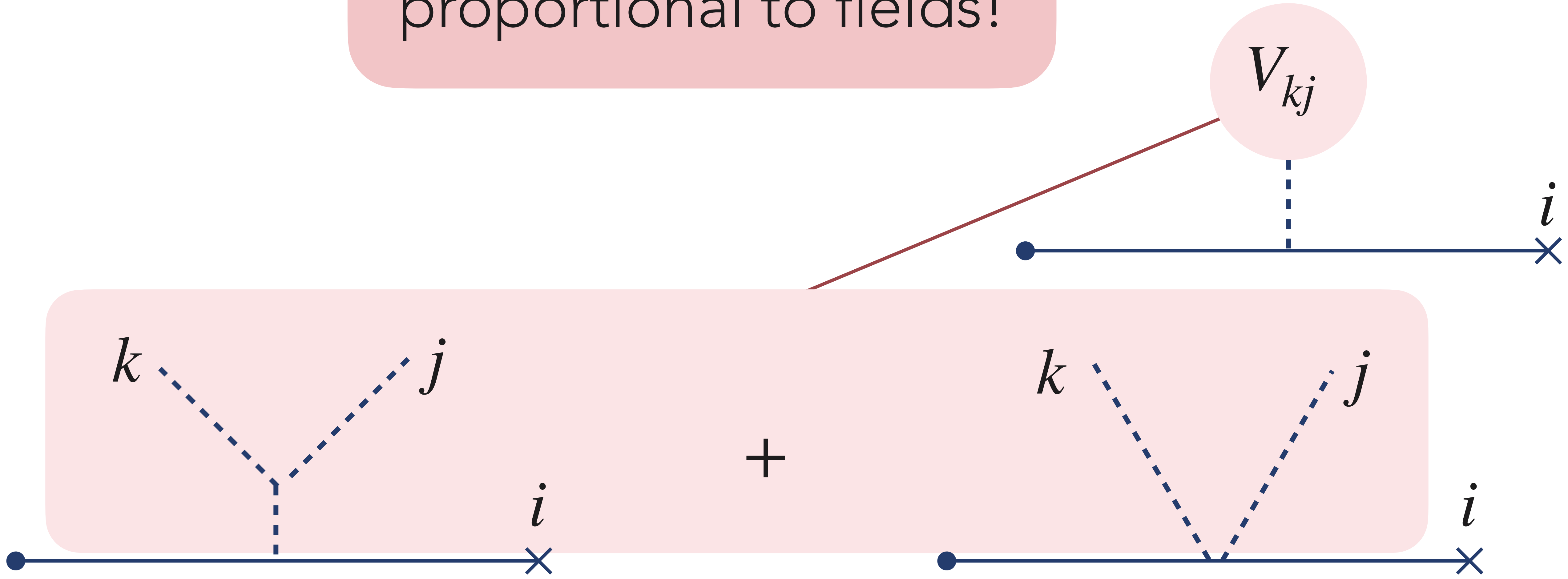


$$V_{kj}(\tau) = -\left(ig\phi_{kj} + i\lambda\phi_k\phi_j\right)\Big|_{X(\tau)}$$

$$V^{(n)}(\tau) = -\left(ig\phi^{(n)} + \frac{i\lambda}{2}\sum_{m+l=n}\binom{n}{m}\phi^{(m)}\phi^{(l)}\right)\Bigg|_{X(\tau)}$$

Same recursion holds!

General polynomial scalar potentials

Order n interaction

$$\mathcal{U}(\phi) = \sum_{n=0}^{N} \mathcal{U}_n \qquad \mathcal{U}_n = \frac{\lambda_n}{n!}\phi^n$$

Contribution to vertex operator

$$V_n^{(r)}(\tau) = -\frac{i\lambda_n}{(n-2)!} \sum_{p_1+\ldots+p_{n-2}=r} \binom{r}{p_1,\ldots,p_{n-2}} \phi^{(p_1)}(X(\tau))\ldots\phi^{(p_{n-2})}(X(\tau))$$

$$V^{(r)} = \sum_{m=1}^{r} V_m^{(r)}$$

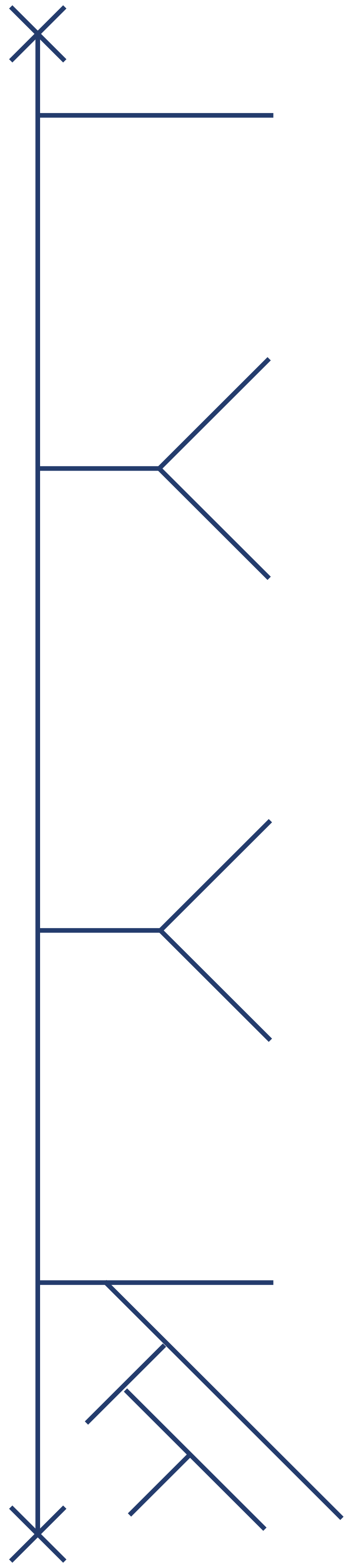
Same recursion holds!

Tree level amplitude recipe

Use **infinite NN** worldline → Choose **asymptotic states**, create with linear VOs → All diagrams from inserting **all possible gVOs** giving N-2 particles → **Fix 1 gVO**: translation symmetry

$$\mathscr{A}_N \propto (2\pi)^D \delta^D\Big(\sum_{j=1}^{N} k_i\Big)\Big\langle\!\Big\langle \phi_1(X(-\infty)) \sum_{m=1}^{N-2} \sum_{I_1 I_2 \cdots I_m = 23\cdots N-1} V_{I_1}(0) \prod_{k=2}^{m} \int_{-\infty}^{+\infty} d\tau_k \, V_{I_k}(\tau_k)\, \phi_N(X(+\infty)) \Big\rangle\!\Big\rangle$$

Overview



Motivation

Scalar worldline path integrals

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Multiparticle fields and generalized vertex operators

General scalar trees

Summary & Outlook

In short:

QFT amplitudes=Worldlines of various geometries

Summary

Propagators
Finite DD worldline

Introduced formalism
✓ **Worldline** based
✓ Implementing **equivalent** external legs

Multiparticle fields
Semi-infinite DN worldline

Vertex Operators
Pullbacks of fields to worldline

Generalized to **multilinear** case

Obtained **recursive** relation

Amplitudes
Infinite NN worldline

Full N-point amplitude as worldline correlator

Asymptotic states via vertex operators

Make all possible remaining gVO insertions

Outlook

How to extend framework to **more general** theories? **Gauge theory? Gravity?**

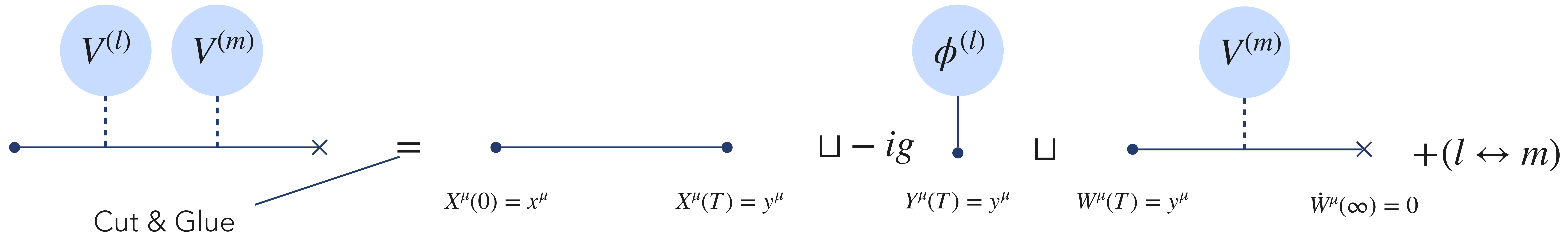
Extend specifically to **Yang Mills:** Investigate kinematic algebra from **Vertex Operator Algebra**

Hope to get insights on **Color-Kinematics duality**

Thank you for your attention!

Deriving the recursion

Term contributing to the sum:



Cut & Glue preserving number of moduli

$$= -ig \int d^D y Z_0(x,y) \phi^{(l)}(y) \left\langle \bar{V}^{(m)} \phi^{(1)}(X(\infty)) \right\rangle_{DN}^{\infty} + (l \leftrightarrow m)$$

Idea

Isolate earliest gVO insertion, sum over all possibilities → Cut and glue at earliest insertion time → Sum over lower order gVOs appears

➜ Recover same multilinear fields as from perturbative solutions