RTG 2575:
Rethinking Quantum Field Theory

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Worldline Geometries for Scattering Amplitudes

Based on work in collaboration with Roberto Bonezzi[1]

Motivation

In this work, we formulate amplitudes in QFT as **worldline path integrals**.

Why?
- **Efficiency:** one worldline diagram corresponds to many Feynman diagrams in perturbation theory.
- **Color-Kinematics duality:** The double copy structure of gravity is best understood in the string formulation, so equivalently for QFT we can expect it to show up in the worldline formalism.

Ultimate goal: Understand **algebraic structure** underlying color-kinematics duality!

Setup

Toy model: Open scalar worldline

$$S_0[X,P,e] = \int_0^1 d\tau \left[P_\mu \dot{X}^\mu - e\left(P^2 + m^2\right) \right]$$

$\tau = 0$ $\qquad$ $\tau = 1$

Requires two boundary conditions:
- Dirichlet (D): on position
- Neumann (N): on momentum

Represents free worldline path integral:

$$Z_0(x,y) = \int \frac{\mathscr{D}X \mathscr{D}P \mathscr{D}e}{\text{VolGauge}} e^{iS_0[X,e]}$$

Interactions: Dress with **vertex operators***

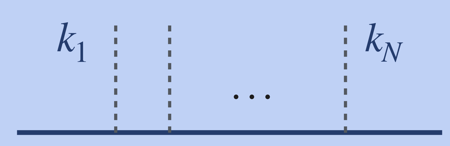
We consider the particle to be interacting with a background field ϕ with a potential $\mathscr{U}(\phi)$. This induces a potential on the particle given by $\mathscr{U}''(X)$.

To define vertex operators, use the potential for a plane wave background:

$$V_i(\tau) := -i\mathscr{U}''(e^{ik_i \cdot X(\tau)})$$

The full path integral comes from summing over all insertions.

At order N:

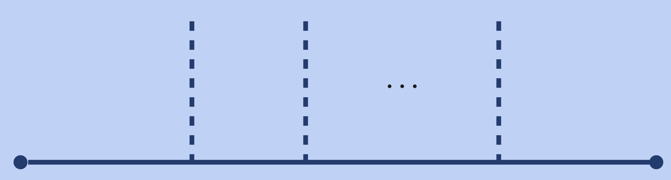


*VO's from now on

Geometries

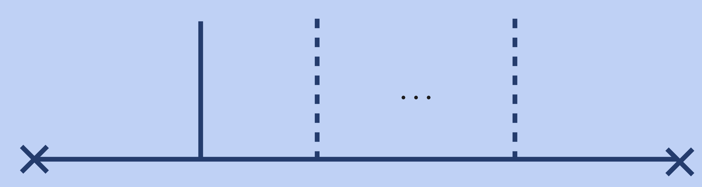
In order to reformulate QFT amplitudes in a worldline based framework, we postulate a dictionary between **QFT objects** and **worldline path integrals** with specific boundary conditions and worldline geometries:

Finite line, DD | **Propagators**



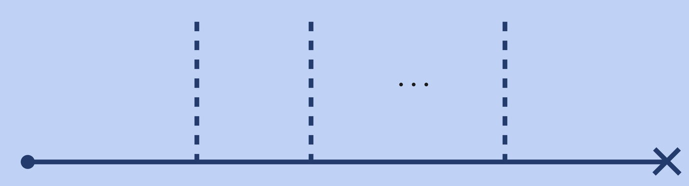
- Gauge fix: $e = T$
- Integrate over T: modulus
- Integrate over positions of VOs

Infinite line, NN | **Amplitudes**



- Gauge fix: $e = 1$
- $T \to \infty$: **LSZ reduction!**
- Create asymptotic states with VOs
- Translation invariance: need to fix one VO position

Semi-infinite line, DN | **VOs**



- Gauge fix: $e = 1$
- $T \to \infty$
- Integrate over positions of VOs
- Related to multiparticle fields

Recursion

Multiparticle fields can be obtained by solving the equations of motion with the ansatz:

$$\phi(x) = \sum_{n=1}^{\infty} \frac{1}{n!} \phi^{(n)}(x)$$

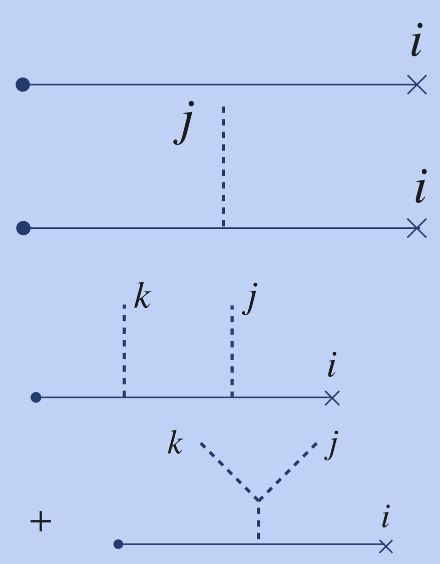
We can also recover these solutions by using **worldline correlators.**

Example: ϕ^3 theory

$\phi_i(x) =$

$\phi_{ij}(x) =$

$\phi_{ijk}(x) =$



In the unpolarized case, for **any polynomial scalar self-interaction,** multiparticle fields are given as worldline correlators through the following **recursive relation:**

$$\sum_{m=1}^{\infty} \frac{1}{(m-1)!} \phi^{(m)}(x) = \left\langle \exp\left(\sum_{k=1}^{\infty} \frac{1}{k!} \overline{V}^{(k)} \right) \phi^{(1)}(X(\infty)) \right\rangle_{\text{DN}}^{\infty}$$

Integrated vertex operators $\qquad$ Asymptotic state

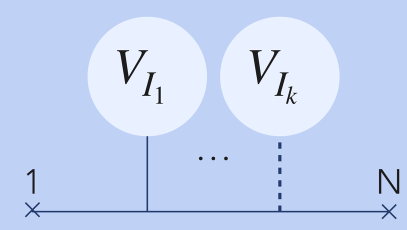
*To polarize and use for finding an N-linear field, substitute

$$\phi_{12\ldots N} = \sum_{n=0}^{N} e^{ik_n \cdot x}$$

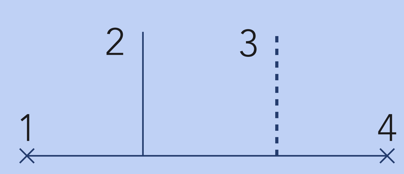
Amplitudes

Here is our recipe for **amplitudes from the worldline:**

Use **infinite NN** worldline → Choose **asymptotic states,** create with linear VOs → Consider all diagrams with all VO insertions giving N-2 particles → **Fix 1 gVO**: translation symmetry



Example: 4-point amplitude in ϕ^3 $\quad \mathscr{A}_4 =$



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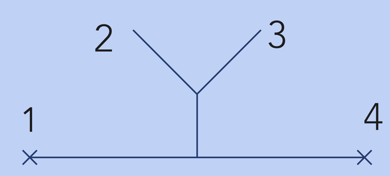

Outlook

By exploring the cutting and gluing of worldlines, we can extend this to **loop level**.

We want to generalize this framework to genuine **gauge theories** [3] and **gravity.**

We are especially interested in **Yang Mills theory**, the **kinematic algebra** of which is related to a **vertex operator algebra.** [2] [4]

Thanks for stopping by!

Poster by: Maria Kallimani

Bibliography:

[1] R. Bonezzi, MFK, *Worldline geometries for scattering amplitudes*
[2] R. Bonezzi, C. Chiaffrino, O. Hohm, *Vertex operators for the kinematic algebra of Yang-Mills theory*
[3] F. Bastianelli, R. Bonezzi, O. Corradini, F. Fecit, *Gluon amplitudes in first quantization*
[4] R. Bonezzi, C. Chiaffrino, O. Hohm, MFK, *Yang-Mills kinematic algebra via homotopy transfer from a worldline operator algebra*